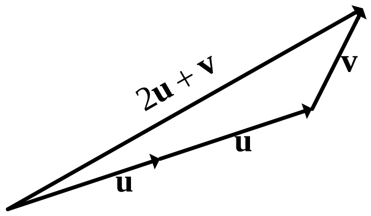
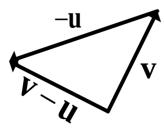
Oppgave 3.1:

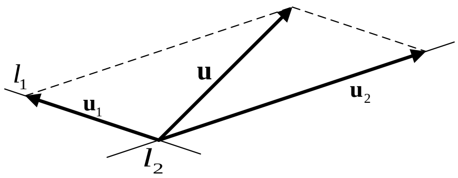
a)

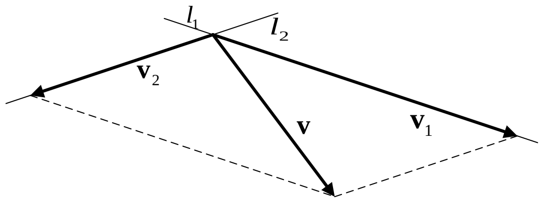


b)



Oppgave 4.1:

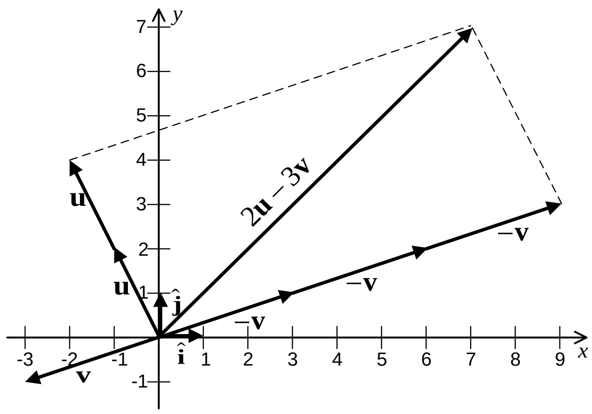




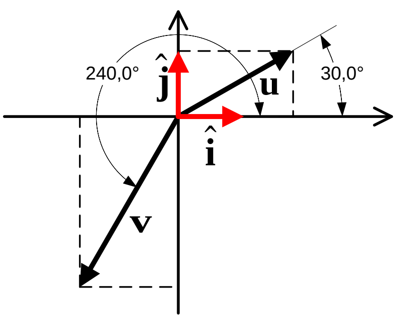
Oppgave 5.1:

a) $2\mathbf{u}-3\mathbf{v}=2\left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)-3\left(-3\hat{\mathbf{i}}-\hat{\mathbf{j}}\right)=-2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+9\hat{\mathbf{i}}+3\hat{\mathbf{j}}=\underline{\underline{7\hat{\mathbf{i}}+7\hat{\mathbf{j}}}}$.

b)



Oppgave 5.2:



Av figuren til venstre ser vi at:

$$u_x=|\mathbf{u}|\cos 30^\circ=2\cdot\tfrac{1}{2}\sqrt{3}=\sqrt{3}\,,$$

$$u_y=|\mathbf{u}|\sin 30^\circ=2\cdot\tfrac{1}{2}=1\,.$$

$$v_x=|\mathbf{v}|\cos 240^\circ=3\cdot\left(-\tfrac{1}{2}\right)=-\tfrac{3}{2}\,,$$

$$v_y=|\mathbf{v}|\sin 240^\circ=3\cdot\left(-\tfrac{1}{2}\sqrt{3}\right)=-\tfrac{3}{2}\sqrt{3}\,.$$

Da blir

$$\mathbf{u}=u_x\hat{\mathbf{i}}+u_y\hat{\mathbf{j}}=\sqrt{3}\hat{\mathbf{i}}+\hat{\mathbf{j}}\,,\qquad \mathbf{v}=v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}=-\tfrac{3}{2}\hat{\mathbf{i}}-\tfrac{3}{2}\sqrt{3}\hat{\mathbf{j}}\,.$$

Dermed blir

$$\mathbf{u}-2\mathbf{v}=\left(\sqrt{3}\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)-2\left(-\tfrac{3}{2}\hat{\mathbf{i}}-\tfrac{3}{2}\sqrt{3}\hat{\mathbf{j}}\right)=\sqrt{3}\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{i}}+3\sqrt{3}\hat{\mathbf{j}}=\underline{\underline{\left(\sqrt{3}+3\right)\hat{\mathbf{i}}+\left(1+3\sqrt{3}\right)\hat{\mathbf{j}}}}\,.$$

Oppgave 6.1:

a) $\mathbf{u}=[2,\,3] \quad \Leftrightarrow \quad |\mathbf{u}|=\sqrt{2^2+3^3}=\sqrt{4+9}=\underline{\underline{\sqrt{13}}}$.

$\mathbf{v}=[-1,\,-1] \quad \Leftrightarrow \quad |\mathbf{v}|=\sqrt{(-1)^2+(-1)^2}=\sqrt{1+1}=\underline{\underline{\sqrt{2}}}$.

$\mathbf{u}\cdot\mathbf{v}=[2,3]\cdot[-1,-1]=2\cdot(-1)+3\cdot(-1)=-2-3=\underline{\underline{-5}}$.

$\mathbf{u}\cdot\mathbf{v}=|\mathbf{u}|\cdot|\mathbf{v}|\cdot\cos\theta \quad \Leftrightarrow \quad \cos\theta=\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|}=\dfrac{-5}{\sqrt{13}\cdot\sqrt{2}}\approx \underline{-0.9806} \quad \Leftrightarrow \quad \theta\approx\underline{\underline{169^\circ}}$.

b) $\mathbf{u}=[-1,\,1,\,3] \quad \Leftrightarrow \quad |\mathbf{u}|=\sqrt{(-1)^2+1^2+3^2}=\underline{\underline{\sqrt{11}}}$.

$\mathbf{v}=[-3,\,2,\,1] \quad \Leftrightarrow \quad |\mathbf{v}|=\sqrt{(-3)^2+2^2+1^2}=\underline{\underline{\sqrt{14}}}$.

$\mathbf{u}\cdot\mathbf{v}=[-1,\,1,\,3]\cdot[-3,\,2,\,1]=(-1)\cdot(-3)+1\cdot 2+3\cdot 1=3+2+3=\underline{\underline{8}}$.

$\mathbf{u}\cdot\mathbf{v}=|\mathbf{u}|\cdot|\mathbf{v}|\cdot\cos\theta \quad \Leftrightarrow \quad \cos\theta=\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|}=\dfrac{8}{\sqrt{11}\cdot\sqrt{14}}\approx \underline{0.6447} \quad \Leftrightarrow \quad \theta\approx\underline{\underline{50^\circ}}$.

Oppgave 6.2:

a) $[2,\,1]\cdot[-1,\,a]=0 \quad \Leftrightarrow \quad -2+a=0 \quad \Leftrightarrow \quad a=\underline{\underline{2}}$.

b) $[a,\,1]\cdot\left[-1,\,2a^2\right]=0 \quad \Leftrightarrow \quad -a+2a^2=0 \quad \Leftrightarrow \quad a(-1+2a)=0 \quad \Leftrightarrow \quad \begin{cases} a=\underline{\underline{0}} \\ a=\underline{\underline{\tfrac{1}{2}}} \end{cases}$

c) $[a,\,-a,\,1]\cdot[2a,\,1,\,-1]=0 \quad \Leftrightarrow \quad 2a^2-a-1=0$

$$a=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 2\cdot(-1)}}{2\cdot 2}=\frac{1\pm\sqrt{9}}{4}=\begin{cases} \frac{1+3}{4}=\underline{\underline{1}} \\ \frac{1-3}{4}=\underline{\underline{-\frac{1}{2}}} \end{cases}$$

Oppgave 6.3:

a) Kaller vektoren $\mathbf{v}=[x,y]$. Da vet jeg:

$|\mathbf{v}|=5 \quad \Leftrightarrow \quad \sqrt{x^2+y^2}=5 \quad \Leftrightarrow \quad x^2+y^2=25$.

$\mathbf{v}$ står vinkelrett på $[2,\,1] \quad \Leftrightarrow \quad [x,y]\cdot[2,1]=0 \quad \Leftrightarrow \quad 2x+y=0 \quad \Leftrightarrow \quad y=-2x$.

Da blir

$x^2+y^2=25 \quad \Leftrightarrow \quad x^2+(-2x)^2=25 \quad \Leftrightarrow \quad x^2+4x^2=25 \quad \Leftrightarrow \quad 5x^2=25$

$\Leftrightarrow \quad x^2=5 \quad \Leftrightarrow \quad x=\underline{\pm\sqrt{5}}$

Får to løsninger:

$[x,y]=\underline{\underline{\left[\sqrt{5},-2\sqrt{5}\right]}}, \qquad [x,y]=\underline{\underline{\left[-\sqrt{5},2\sqrt{5}\right]}}$.

b) Kaller vektoren $\mathbf{v} = [x, y]$. Da vet jeg:

$$|\mathbf{v}| = 4 \quad \Leftrightarrow \quad \sqrt{x^2 + y^2} = 4 \quad \Leftrightarrow \quad x^2 + y^2 = 16.$$

v står vinkelrett på $[1, -1]$ $\quad \Leftrightarrow \quad [x, y] \cdot [1, -1] = 0 \quad \Leftrightarrow \quad x - y = 0 \quad \Leftrightarrow \quad y = x.$

Da blir

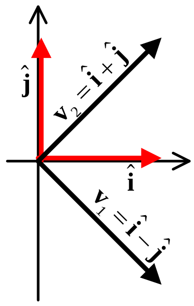
$$x^2 + y^2 = 16 \quad \Leftrightarrow \quad x^2 + x^2 = 16 \quad \Leftrightarrow \quad 2x^2 = 16 \quad \Leftrightarrow \quad x = \underline{\pm\sqrt{8}} = \underline{\pm 2\sqrt{2}}.$$

Får to løsninger:

$$[x, y] = \underline{\underline{\left[2\sqrt{2}, 2\sqrt{2}\right]}}, \qquad [x, y] = \underline{\underline{\left[-2\sqrt{2}, -2\sqrt{2}\right]}}.$$

Oppgave 7.1:

a)



$\mathbf{v}_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ og $\mathbf{v}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}}$.

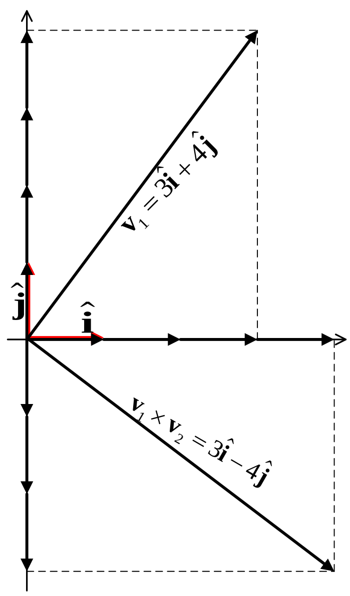
$$\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \hat{\mathbf{k}} = \underline{\underline{2\hat{\mathbf{k}}}}.$$

Vi ser at $|\mathbf{v}_1| = |\mathbf{v}_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$.

Da blir $|\mathbf{v}_1 \times \mathbf{v}_2| = |\mathbf{v}_1| \cdot |\mathbf{v}_2| \cdot \sin\theta = \sqrt{2} \cdot \sqrt{2} \cdot \sin 90^\circ = 2 \cdot 1 = 2$.

Høyrehåndsregelen gir at $\mathbf{v}_1 \times \mathbf{v}_2$ må komme mot oss ut av papirplanet.

b)



$\mathbf{v}_1 = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ og $\mathbf{v}_2 = \hat{\mathbf{k}}$.

$$\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix} \hat{\mathbf{k}} = \underline{\underline{4\hat{\mathbf{i}} - 3\hat{\mathbf{j}}}}.$$

Vi ser at $|\mathbf{v}_1| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$ mens $|\mathbf{v}_2| = 1$.

Siden $\mathbf{v}_1$ ligger i *xy*-planet mens $\mathbf{v}_2$ står vinkelrett ut av dette planet, blir vinkel θ mellom $\mathbf{v}_1$ og $\mathbf{v}_2$ lik 90°.

Dermed blir $|\mathbf{v}_1 \times \mathbf{v}_2| = |\mathbf{v}_1| \cdot |\mathbf{v}_2| \cdot \sin\theta = 5 \cdot 1 \cdot \sin 90^\circ = 5$.

Videre skal $\mathbf{v}_1 \times \mathbf{v}_2$ stå vinkelrett på både $\mathbf{v}_1$ og $\mathbf{v}_2$ slik at høyrehåndsregelen er oppfylt. Da må $\mathbf{v}_1 \times \mathbf{v}_2$ være den vektoren som er vist på figuren.

Oppgave 7.2:

a) En vektor som står vinkelrett på både **u** og **v** er

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 0 \\ 3 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ 1 & 1 \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} \hat{\mathbf{k}}$$

$$= (-2 - 0)\hat{\mathbf{i}} - (1 - 0)\hat{\mathbf{j}} + (1 + 6)\hat{\mathbf{k}} = \underline{-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 7\hat{\mathbf{k}}}$$

En enhetsvektor i denne retningen er

$$\mathbf{e} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 7\hat{\mathbf{k}}}{\sqrt{(-2)^2 + (-1)^2 + 7^2}} = \underline{\underline{\frac{1}{\sqrt{54}}\left(-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 7\hat{\mathbf{k}}\right)}}$$

b) En vektor som står vinkelrett på både **u** og **v** er

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & 1 \\ -1 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}\hat{\mathbf{i}} - \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix}\hat{\mathbf{j}} + \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix}\hat{\mathbf{k}}$$

$$= (4-3)\hat{\mathbf{i}} - (0+1)\hat{\mathbf{j}} + (0+2)\hat{\mathbf{k}} = \underline{\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}}$$

En enhetsvektor i denne retningen er

$$\mathbf{e} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{1^2 + (-1)^2 + 2^2}} = \underline{\underline{\frac{1}{\sqrt{6}}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right)}}.$$

c) En vektor som står vinkelrett på både **u** og **v** er

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -1 \\ 0 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 2 & 3 \end{vmatrix}\hat{\mathbf{i}} - \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix}\hat{\mathbf{j}} + \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}\hat{\mathbf{k}}$$

$$= (6+2)\hat{\mathbf{i}} - (3-0)\hat{\mathbf{j}} + (2-0)\hat{\mathbf{k}} = \underline{8\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}$$

En enhetsvektor i denne retningen er

$$\mathbf{e} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{8\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{8^2 + (-3)^2 + 2^2}} = \underline{\underline{\frac{1}{\sqrt{77}}\left(8\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right)}}.$$