Forkunnskaper i matematikk for fysikkstudenter.
Integrasjon-test løsning.

Oppgave 1:

a) $\int (3x^2-2x+1)dx = 3\int x^2dx - 2\int x\,dx + \int 1dx$

$= 3\cdot\frac{1}{3}x^3 - 2\cdot\frac{1}{2}x^2 + x + C = \underline{\underline{x^3 - x^2 + x + C}}$

b) $\int e^x dx = \underline{\underline{e^x + C}}$ fordi $\frac{d}{dx}e^x = e^x$.

c) $\int (\sin x - 2\cos x)dx = \underline{\underline{-\cos x - 2\sin x + C}}$

d) $\int \frac{2}{x}dx = \underline{\underline{2\ln x + C}} = \underline{\underline{\ln(x^2) + C}}$ fordi $\frac{d}{dx}\ln x = \frac{1}{x}$.

e) $\int \frac{2}{x^2}dx = 2\int x^{-2}dx = 2\cdot\frac{1}{-2+1}x^{-2+1} + C = -2x^{-1} + C = \underline{\underline{\frac{-2}{x}}} + C$.

Oppgave 2: Beregn disse bestemte integralene:

a) $\int_0^2 (x-2)dx = \left[\frac{1}{2}x^2 - 2x\right]_0^2 = \left(\frac{1}{2}\cdot 2^2 - 2\cdot 2\right) - \left(\frac{1}{2}\cdot 0^2 - 2\cdot 0\right) = 2 - 4 - 0 + 0 = \underline{\underline{-2}}$

b) $\int_0^{\ln 3} 2e^x dx = 2\left[e^x\right]_0^{\ln 3} = 2\left(e^{\ln 3} - e^0\right) = 2(3-1) = \underline{\underline{4}}$

c) $\int_0^{\frac{1}{2}\pi} \cos x\,dx = [\sin x]_0^{\frac{1}{2}\pi} = \sin\left(\frac{1}{2}\pi\right) - \sin 0 = 1 - 0 = \underline{\underline{1}}$

d) $\int_1^e \frac{1}{x}dx = [\ln x]_1^e = \ln e - \ln 1 = 1 - 0 = \underline{\underline{1}}$

e) $\int_1^\infty \frac{2}{x^3}dx = 2\int_1^\infty x^{-3}dx = 2\cdot\left[\frac{1}{-3+1}x^{-3+1}\right]_1^\infty = \frac{2}{-2}\left[x^{-2}\right]_1^\infty = -\left[\frac{1}{x^2}\right]_1^\infty = -0 + \frac{1}{1^2} = \underline{\underline{1}}$

Oppgave 3: Du må bruke substitusjon for å løse integralene nedenfor:

a) $\int e^{-2x}dx = \int e^u\left(-\frac{1}{2}du\right) = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = \underline{\underline{-\frac{1}{2}e^{-2x} + C}}$.

Har brukt substitusjonen $u = -2x \quad\Rightarrow\quad \frac{du}{dx} = -2 \quad\Leftrightarrow\quad dx = -\frac{1}{2}du$

b) $\int \sin(3x)dx = \int \sin u\cdot\frac{1}{3}du = \frac{1}{3}\int \sin u\,du = \frac{1}{3}(-\cos u) + C = \underline{\underline{-\frac{1}{3}\cos(3x) + C}}$.

Har brukt substitusjonen $u = 3x \quad\Rightarrow\quad \frac{du}{dx} = 3 \quad\Leftrightarrow\quad dx = \frac{1}{3}du$

c) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = \int \frac{-du}{u} = -\ln u + C = \underline{\underline{-\ln(\tan x) + C}}$.

Har brukt substitusjonen $u = \cos x \quad\Rightarrow\quad \frac{du}{dx} = -\sin x \quad\Leftrightarrow\quad du = -\sin x\,dx$

d) $\int \frac{x}{x^2+4}dx = \int \frac{x}{u}dx = \frac{1}{2}\int \frac{1}{u}\cdot(2x\,dx) = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln u + C = \underline{\underline{\frac{1}{2}\ln\left(x^2+4\right) + C}}$.

Har brukt substitusjonen $u = x^2 + 4 \quad\Rightarrow\quad \frac{du}{dx} = 2x \quad\Leftrightarrow\quad du = 2x\,dx$