**Oppgave 2.1:**

a)
$$\int_1^2 \frac{x^2+x-1}{x}dx = \int_1^2\left(x+1-\frac{1}{x}\right)dx = \left[\frac{1}{2}x^2+x-\ln|x|+C\right]_1^2$$
$$=\left(\tfrac{1}{2}\cdot 2^2+2-\ln 2\right)-\left(\tfrac{1}{2}\cdot 1^2+1-\ln 1\right)=\left(4-\ln 2\right)-\tfrac{3}{2}=\underline{\underline{\tfrac{5}{2}-\ln 2}}$$

b)
$$\int_0^3 (x+2)^2\,dx = \int_0^3\left(x^2+4x+4\right)dx = \left[\tfrac{1}{3}x^3+2x^2+4x+C\right]_0^3$$
$$=\left(\tfrac{1}{3}\cdot 3^3+2\cdot 3^2+4\cdot 3\right)-\left(\tfrac{1}{3}\cdot 0^3+2\cdot 0^2+4\cdot 0\right)=39-0=\underline{\underline{39}}$$

c)
$$\int_1^4 \frac{x-1}{\sqrt{x}}dx = \int_1^4\left(\frac{x}{\sqrt{x}}-\frac{1}{\sqrt{x}}\right)dx = \int_1^4\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx = \int_1^4\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$$
$$=\left[\frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1}-\frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1}+C\right]_1^4=\left[\frac{1}{\frac{3}{2}}x^{\frac{3}{2}}-\frac{1}{\frac{1}{2}}x^{\frac{1}{2}}\right]_1^4=\left[\frac{2}{3}x\cdot x^{\frac{1}{2}}-2x^{\frac{1}{2}}\right]_1^4$$
$$=\left[\sqrt{x}\cdot\left(\tfrac{2}{3}x-2\right)\right]_1^4=\left(\sqrt{4}\cdot\left(\tfrac{2}{3}\cdot 4-2\right)\right)-\left(\sqrt{1}\cdot\left(\tfrac{2}{3}\cdot 1-2\right)\right)=2\cdot\tfrac{2}{3}-1\cdot\left(-\tfrac{4}{3}\right)=\underline{\underline{\tfrac{8}{3}}}$$

**Oppgave 3.1:**

Deloppgave a) og b) løses enklest med formelen $\int e^{ax}dx=\frac{1}{a}e^{ax}+C$.

a) $\int e^{-t}dt=\frac{1}{-1}e^{-t}+C=\underline{\underline{-e^{-t}+C}}$.

b) $\int e^{3t}dt=\underline{\underline{\frac{1}{3}e^{3t}+C}}$.

c) $\int \frac{1}{x-3}dx=\int\frac{1}{u}du=\ln|u|+C=\underline{\underline{\ln|x-3|+C}}$.

Her har vi benyttet substitusjonen
$$u=x-3 \quad\Rightarrow\quad \frac{du}{dx}=1 \quad\Leftrightarrow\quad du=dx\,.$$

d) $\int \frac{2}{3x+1}dx=2\int\frac{1}{u}\cdot\frac{1}{3}dx=\frac{2}{3}\ln|u|+C=\underline{\underline{\frac{2}{3}\ln|3x+1|+C}}$.

Her har vi benyttet substitusjonen
$$u=3x+1 \quad\Rightarrow\quad \frac{du}{dx}=3 \quad\Leftrightarrow\quad dx=\tfrac{1}{3}du\,.$$

e)
$$\int\frac{x}{\sqrt{x^2+4}}dx=\int\frac{\cancel{x}}{\sqrt{u}}\cdot\frac{1}{2\cancel{x}}du=\frac{1}{2}\int u^{-\frac{1}{2}}du=\frac{1}{2}\cdot\frac{1}{-\frac{1}{2}+1}u^{-\frac{1}{2}+1}+C=\frac{1}{-1+2}u^{\frac{1}{2}}+C$$
$$=\sqrt{u}+C=\underline{\underline{\sqrt{x^2+4}+C}}$$

Her har vi benyttet substitusjonen
$$u=x^2+4 \quad\Leftrightarrow\quad \frac{du}{dx}=2x \quad\Leftrightarrow\quad dx=\frac{1}{2x}du\,.$$

f) $$\int x^2\sqrt{x^3+1}\,dx = \int \cancel{x^2}\sqrt{u}\cdot\frac{1}{3\cancel{x^2}}du = \frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\cdot\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1}+C = \underline{\underline{\frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}}+C}}.$$

Her har vi benyttet substitusjonen

$$u = x^3+1 \quad\Leftrightarrow\quad \frac{du}{dx}=3x^2 \quad\Leftrightarrow\quad dx = \frac{1}{3x^2}du.$$

**Oppgave 3.2:**

a) $$\int_1^4 \frac{x}{x^2+9}dx$$

Bruker substitusjonen

$$u = x^2+9 \quad\Rightarrow\quad \frac{du}{dx}=2x \quad\Leftrightarrow\quad dx=\frac{du}{2x}.$$

Ser også at når $x=1$ blir $u=1^2+9=10$, og når $x=4$ blir $u=4^2+9=25$.

Da blir

$$\int_1^4 \frac{x}{x^2+9}dx = \int_{u=10}^{u=25}\frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} = \tfrac{1}{2}\int_{10}^{25}\frac{du}{u} = \left[\tfrac{1}{2}\ln u + C\right]_{10}^{25} = \tfrac{1}{2}\ln 25 - \tfrac{1}{2}\ln 10$$

$$= \ln\left(25^{\frac{1}{2}}\right) - \ln\left(10^{\frac{1}{2}}\right) = \ln\sqrt{25} - \ln\sqrt{10} = \underline{\underline{\ln 5 - \ln\sqrt{10}}} = \underline{\underline{\ln 5 - \tfrac{1}{2}\ln 10}}$$

b) $$\int_0^{\frac{\pi}{2}} \sin x\cdot\cos x\,dx$$

Bruker substitusjonen

$$u=\sin x \quad\Rightarrow\quad \frac{du}{dx}=\cos x \quad\Leftrightarrow\quad dx=\frac{du}{\cos x}.$$

Ser også at når $x=0$ blir $u=\sin 0=0$, og når $x=\frac{\pi}{2}$ blir $u=\sin\frac{\pi}{2}=1$.

Da blir

$$\int_0^{\frac{\pi}{2}}\sin x\cdot\cos x\,dx = \int_{u=0}^{u=1} u\cdot\cancel{\cos x}\cdot\frac{du}{\cancel{\cos x}} = \int_0^1 u\,du = \left[\tfrac{1}{2}u^2+C\right]_0^1 = \tfrac{1}{2}\cdot 1^2 - \tfrac{1}{2}\cdot 0^2 = \underline{\underline{\tfrac{1}{2}}}.$$

c) $$\int_0^2 3x^2\sqrt{x^3+1}\,dx$$

Bruker substitusjonen

$$u=x^3+1 \quad\Rightarrow\quad \frac{du}{dx}=3x^2 \quad\Leftrightarrow\quad dx=\frac{du}{3x^2}.$$

Ser også at når $x=0$ blir $u=0^3+1=1$, og når $x=2$ blir $u=2^3+1=9$.

Da blir

$$\int_0^2 3x^2\sqrt{x^3+1}\,dx = \int_{u=1}^{u=9}\cancel{3x^2}\sqrt{u}\cdot\frac{du}{\cancel{3x^2}} = \int_1^9 u^{\frac{1}{2}}du = \left[\frac{1}{\frac{1}{2}+1}u^{\frac{1}{2}+1}+C\right]_1^9$$

$$= \left[\frac{2}{3}u\cdot\sqrt{u}\right]_1^9 = \frac{2}{3}\cdot\left(9\cdot\sqrt{9} - 1\cdot\sqrt{1}\right) = \frac{2}{3}\left(27-1\right) = \underline{\underline{\frac{52}{3}}}$$